

Introduction to Decision Analysis

***** INTRODUCTION EXAMPLE *****



Figure 1: Apple I On display at the Smithsonian (Credit: Ed Uthman, Creative Commons Attribution-Share Alike 2.0 Generic)

The *Apple I* personal computing system came about in 1976 and was just the beginning of the revolution in personal computing. Starting in the garage of his childhood home in Los Altos, California, co-founder Steve Jobs joined with co-founders Steve Wozniak and Ronal Wayne to incorporate a new company, Apple Computer Inc in 1977. Over the years, decisions had to be made regarding the products to develop, details of the products themselves, cost of the products, how and where to market, among many, many other particulars.

These types of decisions are appropriate and constant for any new business owner of any product or service. These decisions should be made critically, quantifiable and carefully as they are critical to the success (and failure) of the business.

***** INTRODUCTION *****

We all make decisions, all the time, from simple and near-term decisions like, “What we are going to eat today?” or, “What we are going to do this weekend?” to more complex long-term decisions involving a detailed thought process and plan like, “How much inventory should be produced?” or, “Should we expand our business with a new store?” Good decisions are based on logic and systematic consideration of the different alternatives at different conditions of the market or environment.

In this unit, you will use **decision theory** to methodically identify and quantify a decision process with decision tables and trees clearly classifying the different options available and estimating potential losses or gains. As a Quantitative Analyst, you will attach numeric values (often in terms of \$) to decision alternatives so that your decision is based not on gut instincts or just previous experience, but on estimated and calculated numbers, whether it is a rating or a monetary amount. Furthermore, you can include probability in your decision-making process to consider an even more informed decision analysis. These are the ideas and methods that multi-billion-dollar companies employ when making informed decisions!

Decision Theory

Decision theory is a general approach to systematically outline your choices and monetary considerations for each choice.

Let us consider the case of Elias Papadimitrikopolous (or Papa Elias as most people call him) who owns and manages his own business selling Greek bakery goods. He started his business about 3 years ago baking and selling his baked goods out of his home in the outskirts of Boulder, Colorado. It has been profitable but now, Papa Elias is considering expanding his little bakery shop.

When decisions are needed, it is helpful to break down the decision process into multiple steps. There are six steps involved in **Decision making**.

The Six Steps in Decision Making:

1. Clearly **define the problem**, isolating the decision you need to make.
2. List the **alternatives**, or choices available in your decision.
3. Identify the possible **states of nature**.
4. **Estimate the potential payoffs** (profit or loss) for each alternative and state of nature combination.
5. Decide on the decision theory **environment and strategy**.
6. Apply the model and make the decision.

(Adapted from Render et al, 2015)

We can consider Papa Elias' situation as we learn the six steps of decision making.

Step 1: The situation that Papa Elias is faced with concerns whether or not he should expand his business by opening a new store in the heart of downtown Boulder.

Step 2: The next step for Papa Elias is to identify what options or courses of action he has for expanding his business in downtown Boulder. Papa Elias will generate the **alternatives** available to him as (1) rent property on Pearl Street, the main pedestrian mall in Boulder (2) sells his goods on a mobile vending cart on Pearl Street or (3) no expansion at all.

It is important to consider all possible alternatives including the “do nothing” alternative even if it has little value. When applying the different decision strategies, it may turn out to be the best course of action to just “stand your ground!”

Now that Papa Elias has identified what alternatives he has, he will want to consider what could potentially happen given each alternative. Well, as we all know, our best laid plans are often affected by the weather! In decision analysis, we call these different “weather conditions”, **states of nature** – they are the outcomes for which the decision maker has little control over but that greatly affect the result of the decision.

Step 3: Papa Elias might consider three different states of nature – a high demand for authentic Greek baked goods, a moderate demand for these goodies, and a low demand for quality home-made treats. Note, that you may only have two states of nature (a good market or bad market) that you would like to consider. The number of states of nature is dependent on how much detail is needed in your decision situation.

Now, comes the critical part of estimating the potential payoffs (profits or losses). This can be done in terms of monetary amounts or perhaps even a rating system. However it is appropriate to measure the benefit for your decision is acceptable.

Step 4: Papa Elias can review his 3 years of experience to make some estimates for how much \$ he might profit or loss given the alternative and state of nature. The objective for Papa Elias' decision is to maximize his profit! He will need to make some estimates for each combination of the alternative and state of nature. Since he has 3 alternatives and 3 states of nature, there will be 9 estimates he will make. He can consider the profit or losses for a week's worth of business but any time frame can be considered as long as it is clearly specified and done for all payoff combinations.

This can be organized with a decision table or sometimes called a payoff table, shown in Figure 2. All of the alternatives are listed as rows and then states of natures are listed as columns. You can see there are 9 combinations to estimate the payoffs for.

Alternatives	State of Nature		
	High Demand	Moderate Demand	Low Demand
Open shop on Pearl	20,000	10,000	-10,000
Vending cart on Pearl	7,500	3,000	-1,000
No expansion at all	0	0	0

Figure 2: Payoff Table for Papa Elias' Greek Bakery in \$ for a Week

Step 5&6: Once the decision table is created, Pap Elias will need to select a decision strategy based on the type of environment that is available to him. These will be explained further in the next sections.

Key Terms

- **Alternative** = The course of actions or choices that the decision maker is considering.
- **State (of Nature)** = The potential future conditions of the environment in which one is making the decision. Some examples might good, fair or poor market; high, moderate or low demand. These conditions the decision-maker has little to no control over but affect the decision to be made.
- **Payoffs** = The conditional values (usually monetary potential profits or losses) attached to the conditional situation that an alternative was chosen under a certain state of nature.
- **Decision/Payoff Table** = A tabular representation of the decision showing payoffs for each combination of alternatives and states.
- **Decision Tree** = A graphical representation of decision making process table showing payoffs for each combination of alternatives and states.

***** **DECISION ENVIRONMENTS** *****

The type of environment that one makes a decision in depends on the amount and type of information known about the situation. Three different **decision environments** are typically considered and will be explained here:

More Key Terms

- **Decision Strategy** (or sometimes called Decision Criteria) = The method used to calculate the decision. For example, Maximax, Maximin, Criteria of Realism.
- **Decision Environment** = The situation in which you are making a decision based on the type and amount of information known. Typically, three decision environments are considered:
 1. **Decision Making Under Certainty**
 2. **Decision Making Under Uncertainty**
 3. **Decision Making Under Risk**

Making a decision when the consequences of each alternative are known with certainty before making the decision is called **Decision Making Under Certainty**. For example, if we are considering different financing options for a new washer and dryer. The local appliance store will give a loan rate 5.5%, your bank will give you a loan rate of 8.7%. All things being equal and fair, we can decide which loan to choose because the outcomes are known for certain.

The more interesting decisions for decision theory involve situations where the outcomes are not certain and the states of nature will be identified and considered in a systematic way with quantitative analysis applied.

For example, when deciding on whether to expand your business, you may not know what the economy will do nor what the demand might be like for your product or services! The next two sections will cover in detail Decision Making Under Uncertainty and Decision Making Under Risk.

***** **DECISION MAKING UNDER UNCERTAINTY** *****

There are several decision strategies when making decisions under uncertainty. We will consider three popular strategies as an introduction to decision theory. For all three methods, we will analyze each alternative (row of decision table) and consider the payoffs for each state of nature (column of decision table).

Strategy

Maximax is an optimistic approach – also called aggressive as it is seeking out the best possible payoff amongst all possible alternatives. This strategy is the “**best of the best**”!

Method – Each alternative (row) in the decision table is considered and the best payoff for each alternative is selected, and recorded in the Row Max column. The maximum of the best payoffs for each alternative then is selected as the maximax decision.

The first strategy that we will consider for Papa Elias’ Greek Bakery is the optimistic approach or **maximax** strategy which considers the best (maximum) payoff for each alternative and then the best (maximum) of these outcomes. It is often called the “best of the best”!

We would review each alternative (row) in the payoff table selecting the maximum of that alternative. Then reviewing the maximums for each alternative, you would select the maximum of these!

Alternatives	State of Nature			Maximum
	High Demand	Moderate Demand	Low Demand	
Open Shop on Pearl	20,000	10,000	-10,000	20,000
Vending Cart on Pearl	7,500	3,000	-1,000	7,500
No expansion at all	0	0	0	0

Using the **maximax** strategy, Papa Elias should expand his business by opening up a new shop directly on Pearl Street so that he could sell his Greek baked delicacies right in downtown on the pedestrian mall! He has a potential payoff to profit by \$20,000 per week! (This is important as rent is really high on Pearl Street!)

Strategy

Maximin is a pessimistic approach – the decision is based on the “best of the worst” as it selects the best amongst the worst payoffs for each alternative. This strategy is the “**best of the worst**”!

Method – Each alternative (row) in the decision table is considered and the worst payoff for each alternative is selected, and recorded in the Row Min column. The maximum of these worst payoffs for each alternative then is selected as the maximin decision.

The second strategy to be considered is the pessimistic approach or **maximin** strategy which considers the worst (minimum) payoff for each alternative and then the best (maximum) of those outcomes. It is often called the “best of the worst”!

We would review each alternative (row) in the payoff table selecting the minimum of that alternative. Then reviewing the minimums for each alternative, you would select the maximum of these!

Alternatives	State of Nature			Minimum
	High Demand	Moderate Demand	Low Demand	
Open Shop on Pearl	20,000	10,000	-10,000	-10,000
Vending Cart on Pearl	7,500	3,000	-1,000	-1,000
No expansion at all	0	0	0	0

Using the **maximin** strategy, Papa Elias should not expand his business by opening an additional shop or with a vending cart on Pearl Street. He would be better off to continue selling his baked goods from his home.

Strategy

Hurwicz Criterion (Criterion of Realism or Hurwicz Criterion) is a compromise between the maximax (optimistic) and maximin (pessimistic) approaches. This strategy is a “**realistic compromise between the best and worst**”.

Method – The decision maker will first need to determine the weight, called the “**coefficient of realism**”, α , for the weighted average calculation. It will be a value between 0 and 1.

The best and worst payoffs for each alternative are selected and used to determine the weighted average for each column using:

$$\text{Hurwicz Criteria} = \alpha (\text{best in row}) + (1 - \alpha) (\text{worst in row})$$

The coefficient of realism, is a measure of the degree of optimism of the decision makers. It will always be between 0 and 1, which can be thought of as 0% to 100%.

- $\alpha = 0$ means the decision be calculated as 0% optimistic, 100% pessimistic
- $\alpha = 0.5$ means the decision be calculated as 50% optimistic, 50% pessimistic
- $\alpha = 1$ means the decision be calculated as 100% optimistic, 0% pessimistic

This strategy allows the decision maker the ability to compromise between the best of the best and the best of the worst. The decision maker can use some subjectivity and personal feelings about the decision.

For Papa Elias, we will already know the best and the worst payoffs for each alternative, as they were calculated in the added column to the payoff table. How about we consider an α of 75% = 0.75. We can now calculate the Hurwitz value for each alternative:

$$\text{Opening a Shop on Pearl} = 0.75(20,000) + (0.25)(-10,000) = \$12,500$$

$$\text{Vending Cart on Pearl} = 0.75(7500) + (0.25)(-1,000) = \$5375$$

$$\text{Do Nothing} = 0.75(0) + (0.25)(0) = \$0$$

Alternatives	State of Nature			Hurwicz
	High Demand	Moderate Demand	Low Demand	
Open Shop on Pearl	20,000	10,000	-10,000	\$ 12,500
Vending Cart on Pearl	7,500	3,000	-1,000	\$ 5,375
No expansion at all	0	0	0	\$ 0

Using the **Hurwicz Criterion** strategy with $\alpha = 0.75$, Papa Elias should expand by opening up a new shop directly on Pearl Street! Again, this would be best so that he could sell his Greek baked delicacies right in downtown on the pedestrian mall with a potential payoff to profit by \$12,500 per week!

We can easily review this decision using a different coefficient of realism as well. Perhaps Papa Elias is not so optimistic and he would rather calculate the decision based on an $\alpha = 0.30$, only 30% optimistic as well. Here are those calculations:

$$\text{Opening a Shop on Pearl} = 0.30(20,000) + (0.70)(-10,000) = -\$1000$$

$$\text{Vending Cart on Pearl} = 0.30(7500) + (0.70)(-1,000) = \$1550$$

$$\text{Do Nothing} = 0.30(0) + (0.70)(0) = \$0$$

Alternatives	State of Nature			Hurwicz
	High Demand	Moderate Demand	Low Demand	
Open Shop on Pearl	20,000	10,000	-10,000	-1,000
Vending Cart on Pearl	7,500	3,000	-1,000	1,550
No expansion at all	0	0	0	0

Now, this certainly changes things doesn't it? Using an 0.30 (only 30% optimistic), Papa Elias' decision now would be to expand only with a vending cart on the walking streets of the pedestrian mall of Pearl Street. This might be a safer approach if the state of nature might learn towards a moderate or low demand for the Greek yummys!

***** **DECISION MAKING UNDER RISK** *****

Decision Making Under Risk is an environment in which the probabilities for the states of nature are known and can be used to better inform the decision process. For example, perhaps we have marketing/survey data that tells us that the economy is looking up and the ability for consumers to spend money on home-made goods of high quality (the type of goods that Papa Elias sells!) If this information is known or can be found, this probability information can be added into the decision process to make an even more informed decision.

As an introduction to this decision environment, we will explore one of the most popular methods which selects the alternative with the highest expected monetary value.

Expected Monetary Value (EMV) is the weighted sum of all possible payoffs for each alternative. It is the *expected value*, or *mean value*, of the long-term average for each alternative. (This will be further discussed in Chapter 7 Discrete Probability Distributions.)

Prior **probabilities** are the probabilities associated with each state of nature. These can be determined by market research, historical trends, or subjective estimation.

This is the Maximum EMV strategy.

Strategy

Expected Monetary Value (EMV) is the weighted sum of all possible payoffs for each alternative.

$$EMV(\text{alternative}) = \sum x_i P(x_i) \text{ where:}$$

x_i = payoff for the alternative in state of nature i

$P(x_i)$ = probability of state of nature i

Method: Once the EMVs for each alternative are calculated, the decision maker will select the maximum EMV from all alternatives.

For example, if the probability for high demand market was 50%, moderate demand was 30% and low demand was 20% we would get:

$$\text{EMV}(\text{Open Shop on Pearl}) = (\$20,000)(0.50) + (\$10,000)(0.30) + (-\$10,000)(0.20) = \$11,000$$

$$\text{EMV}(\text{Vending Cart on Pearl}) = (\$7,500)(0.50) + (\$3,000)(0.30) + (-\$1,000)(0.20) = \$4,450$$

$$\text{EMV}(\text{No expansion at all}) = (\$0)(0.50) + (\$0)(0.30) + (-\$0)(0.20) = \$0$$

Expected Monetary Value (EMV) Decision Process				
Probabilities for State of Nature	0.50	0.30	0.20	
Open Shop on Pearl	20,000	10,000	-10,000	\$ 11,000
Vending Cart on Pearl	7,500	3,000	-1,000	\$ 4,450
No expansion at all	0	0	0	\$ 0
				EMV
				\$ 11,000

Expected Value of Perfect Information

Let us discuss one more aspect of using probability information for our decision analysis. Businesses that need to make decisions often will have to pay for this probability information. Whether it is devoting their own staff to researching the states of environment or outsource to marketing firms that specialize in the technical analysis of the environments. So, this probability information will come at a price. How much should a business pay for this information?

EVPI

Expected Value of Perfect Information (EVPI) is the maximum amount of money that should be paid for the probability information. It is the upper bound for which this probability information will assist with the decision process.

EVPI = expected payoff under certainty – expected payoff under risk or

(EVPI = EVwPI – Max EMV)

The **expected payoff under risk** is the **maximum EMV** that was explained earlier.

The **expected payoff under certainty** (also called **Expected Value with Perfect Information, EVwPI**) is calculated as:

EVwPI

Expected Value with Perfect Information (EVwPI) is the expected or average payoff assuming we have the probability information when the decision is made.

$$EVwPI = \sum (\text{best payoff in state of nature } i)(\text{prob of state of nature } i)$$

Method: Consider each state of nature (column in decision table) and select the maximum payoff for that state. Multiply this to the probability of that state happening.

In the case of Pape Elias' Greek Bakery, we will look at the state of nature of high demand (first column). The best payoff for this column is to open a shop on Pearl Street with a payoff of \$20,000. For moderate demand, the best payoff is also to open a shop on Pearl Street with a payoff of \$10,000. For low demand, the best payoff would be to not expand at all, with a payoff of \$0. These are highlighted below:

Alternatives	State of Nature		
	High Demand	Moderate Demand	Low Demand
Open Shop on Pearl	20,000	10,000	-10,000
Vending Cart on Pearl	7,500	3,000	-1,000
No expansion at all	0	0	0
Probabilities	0.5	0.3	0.2

Then multiplying these payoffs to the associated probabilities for each state of nature, we get:

$$EVwPI = (\$20,000)(0.5) + (\$10,000)(0.3) + (\$0)(0.2) = \$13,000$$

The expected or average return knowing for certain the probability information then is \$13,000.

Using this and the max EMV, we get the Expected Value of Perfect Information as:

$$EVPI = \$13,000 - \$11,000 = \$2,000$$

This tells us that we should pay up to \$2,000 per week get probability information before making our decision!

***** DECISION TREES *****

Another way in which to organize and visualize a decision process is with a decision tree. Decision trees are particularly useful when the situation involves a sequence of decisions towards a final decision.

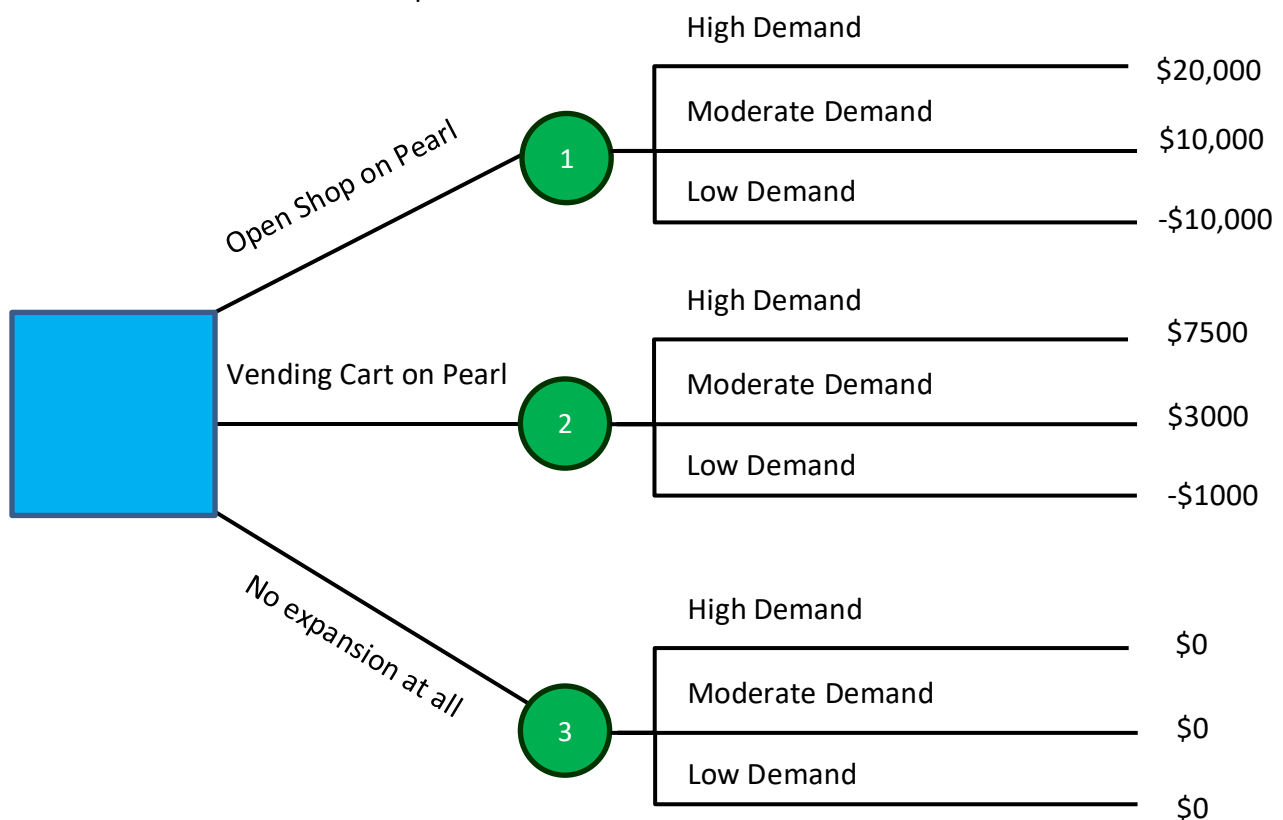
Decision Tree

The decision tree will be constructed using:

- **Decision points** (decision nodes) are drawn with squares. Lines or branches from these represent the alternatives. 
- **State-of-nature points** (state-of-nature nodes) are drawn with circles. Lines or branches from decision nodes represent the states of nature. 

The tree will be drawn and read from left to right presenting the sequential decisions from left to right.

We can visualize the case of Papa Elias' Greek Bakery as:

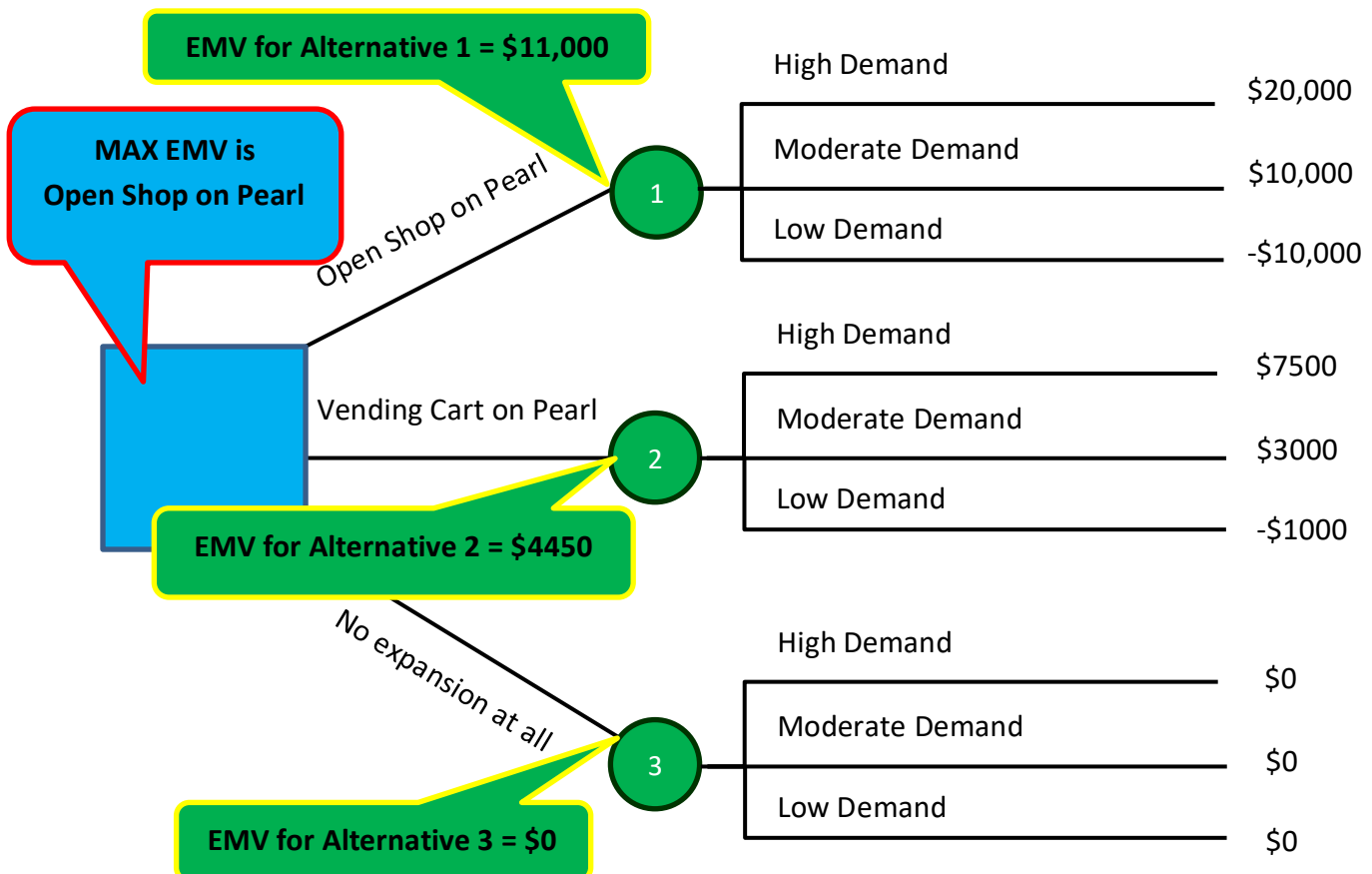


Five Steps for Decision Tree Analysis:

1. Define the problem.
2. Structure or draw the decision tree.
3. Assign probabilities to the states of nature.
4. Estimate payoffs for each possible combination of alternatives and states of nature.
5. Solve the problem by computing EMVs for each state-of-nature node. This is done by working backward, that is, starting at the right of the tree and working back to decision nodes on the left. Also, at each decision node, the alternative with the best EMV is selected.

(Adapted from Render et al, 2015)

In the case of Papa Elias' Greek Bakery, we will apply decision tree analysis:



In our example, there is only one decision node and we will select the alternative that has the best EMV at its state of nature node. In this case, it is to open a shop on Pearl Street!

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